Module 4

POINT SOURCES AND ARRAYS

Structure

- Introduction
- objectives
- Power theorem
- Field patterns
- Array of two isotropic point sources
- Array of n isotropic point sources
- Outcomes
- Questions
- Further Readings

4.0 Introduction

As of now the antenna was treated aperture. in this chapter it is formally considered as point source and later the concept extended to the formation of arrays of point sources. The pattern of any antenna can be regarded as produced by an array of point sources.

Here we discuss the array arrays confined to isotropic point sources, which may represent different kind of antennas.

Point Sources:

***** Antenna that doesn't have any specified shape is called "point source".

Consider an antenna and observation circle as shown in fig.2.1 where the radiated fields of antenna transverse radially at a sufficient distance id far field whereas near fields have actual variation ignored.

Provided that observation made at the sufficient distance, any antenna regardless of size or complexity can be represented as single point source. Far field is considered because power flow and fields are radiated outwards at this region properly.

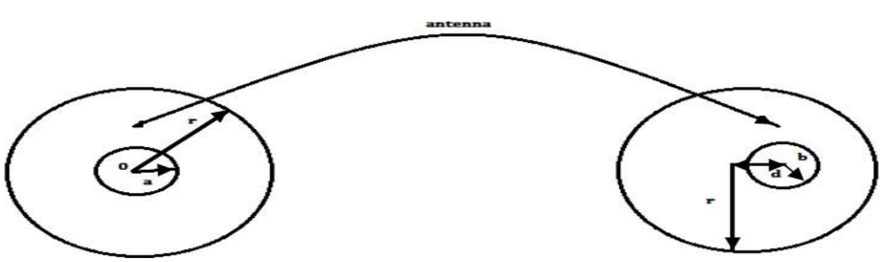


Fig 4.1. Antenna and the observation

Field measurements can be done either by fixing antenna or fixing observation point but both effects are approximately same.

If in case the center of the antenna is displaced by distance 'd' as shown in fig.1, the distance between two centers are negligible effect on the field pattern at observation circle provided that

R >> **d , R** >> **b** and **R** >> λ

As we discussed complete description of the far field of a source requires 3 components.

1. $E_\theta(\theta, \phi)$

2. $E_\phi(\theta, \phi)$

3. $\delta(\theta, \phi)$

Power Patterns:

Let transmitting antenna in free space by point source radiation located at origin of the co-ordinates as shown in fig.4.2.

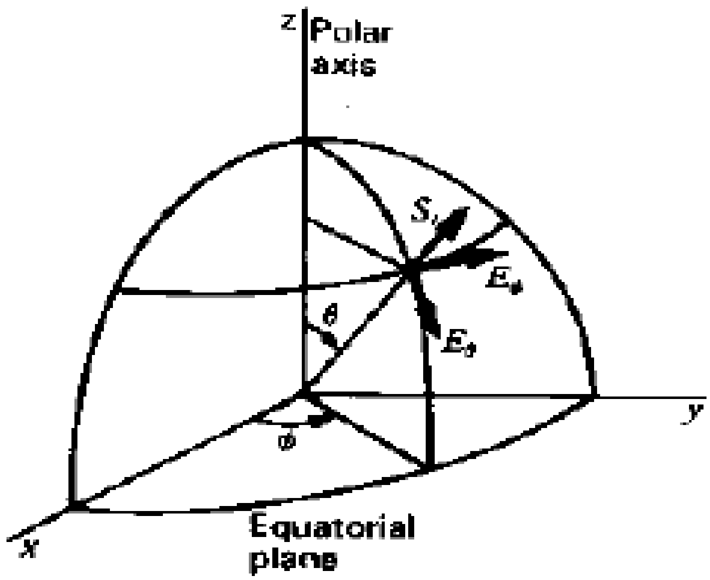


Fig 4.2 Point source at origin.

The radiated energy streams from the source radial lines.

The time rate of energy flow per unit area is "**Poynting vector or power flow density**". The magnitude of Poynting vector is equal t radial component (**|S| = S_r**)

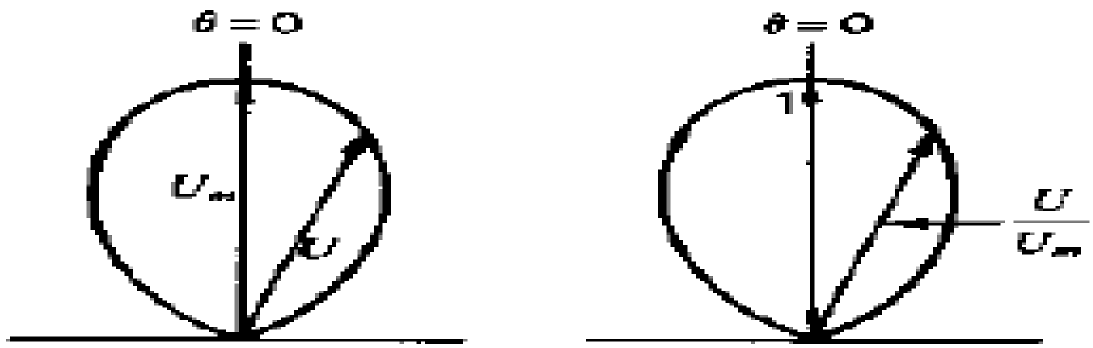
* A sources that radiates energy uniformly in all directions is called "**isotropic antenna**".

A graph of S_r at constant radius as a function of angle is Poynting vector, power density, pattern but usually called "**power pattern**".

Although the isotropic source is convenient in theory, it is not physically realizable type.

Even simplest antenna has bidirectional properties i.e., they radiate energy in some directions than others.

- In contrast with isotropic antennas, they might be called as "**anisotropic antennas**".
- If S_r is expressed in W/m^2 , the absolute power pattern. On the other hand if it express with its reference value then the graph is called "**relative power pattern**".



S_{rm} - maximum power in the direction.

A pattern with a maximum of unity is called "**normalized pattern**".

Objectives

- Apply the power theorem to solve the problems
- Analyse the arrays of point source and their patterns
- Analyse the different conditions and importance of various types of arrays

Power Theorem

If the Poynting vector is known at all points on a sphere of radius **r** from a point source in a lossless medium, the total power radiated by the source is the integral over the surface of the sphere of the radial components **S_{ref}** the average poynting vector". Thus

$$\mathbf{P = \oiint S \cdot ds}$$

Where , P - power radiated (**W**)

S_r - radial component of average poynting vector (Wm^{-2})

ds - infinitesimal element of area of sphere

$$ds = r^2 \sin\theta \, d\theta \, d\phi \ (m^2)$$

For an isotropic source, S_r is independent of **θ and ϕ** so

$$P = S_r \oiint ds$$

$$S_r = \frac{P}{4\pi r^2}$$

This equation indicates that the magnitude of Poynting vector varies inversely proportional to the square of the distance from point source radiator.

Radiation Intensity:

It is defined as the power radiated by an antenna per unit solid angle. Denoted by **u** and unit is W/Sr.

Power Theorem for Radiation Intensity:

The total power radiated by an antenna is given by integral of radiation intensity over solid angle of the sphere.

i.e., $$P = \oiint u \, d\Omega$$

For an isotropic source radiation intensity remains same at any point or the surface of the sphere.

Let u_o be the radiation of isotropic source then

$$P = \oiint u_o \, d\Omega$$

$$P = \oiint u_o \sin\theta \, d\theta \, d\phi$$

$$P = u_o \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$P = 2\pi u_o(-\cos\pi - (-\cos 0))$$

$$P = u_o (4\pi)$$

$$u_o = \frac{P}{4\pi}$$

Therefore, the relation between Poynting vector and radiation intensity as follows

W.K.T. $$S_r = \frac{P}{4\pi r^2}$$

$$S_r r^2 = \frac{P}{4\pi}$$

$$S_r r^2 = u_o$$

$$u_o = S_r r^2$$

Examples Of Power Patterns:

1. Unidirectional Cosine Pattern

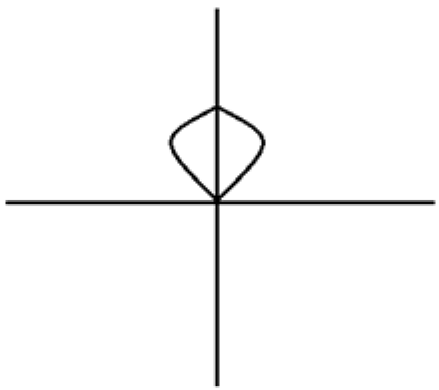
The radiation intensity of unidirectional cosine pattern is given as

$$u = u_m \cos\theta$$

where u_m is the maximum radiation intensity and u is having value in upper hemisphere.

i.e.,

$$\mathbf{u} = \begin{cases} \mathbf{u_m \cos\theta}\ ;\ \mathbf{0} < \theta < \frac{\pi}{2} \\ \qquad\qquad \mathbf{0} < \theta < \boldsymbol{\pi} \\ \mathbf{0};\ \boldsymbol{Elsewhere} \end{cases}$$



2. Bidirectional Cosine Pattern

The radiation intensity of unidirectional cosnine pattern is given as

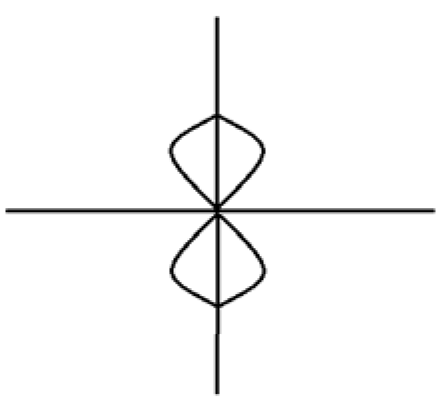
$$u = u_m \cos\theta$$

and has the value in both the hemisphere

i.e.,

$$\mathbf{u} = \begin{cases} \mathbf{u_m \cos\theta}\ ;\ \mathbf{0} < \theta < \boldsymbol{\pi} \\ \qquad\qquad \mathbf{0} < \theta < \boldsymbol{\pi} \\ \mathbf{0};\ \boldsymbol{Elsewhere} \end{cases}$$

It is also known as groundnut pattern because of its appearance as shown.

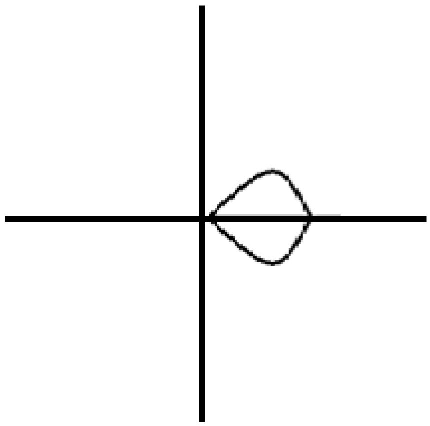


3. Unidirectional Sine Pattern

The radiation intensity is given as

i.e., $$u = \begin{cases} u_m \sin\theta\,; & 0 < \theta < \frac{\pi}{2} \\ & 0 < \theta < \pi \\ 0\,; & Elsewhere \end{cases}$$

The maximum radiation intensity at $\theta = \frac{\pi}{2}$

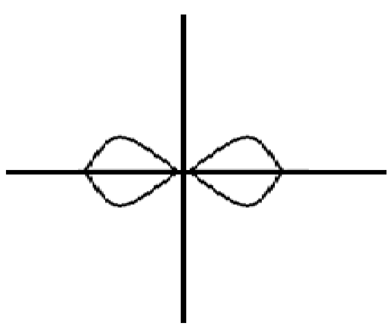


4. Bidirectional Sine Pattern

The radiation intensity is given as

i.e., $$u = \begin{cases} u_m \sin\theta\,; & 0 < \theta < \pi \\ & 0 < \theta < \pi \\ 0\,; & Elsewhere \end{cases}$$

It is also known as doughnut pattern and pattern as shown.



2.3 Field pattern

A pattern showing variation of the electric field intensity at a constant radius r as a function of angle(θ , φ) is called **"field pattern"**

The power pattern and the field patterns are inter-related:

$$P(\theta, \varphi) = (1/\eta)*|E(\theta, \varphi)|2 = \eta*|H(\theta, \varphi)|2$$

P = power
E = electrical field component vector
H = magnetic field component vector
η = 377 ohm (free-space impedance)

The power pattern is the measured (calculated) and plotted received power: |P(θ, φ)| at a constant (large) distance from the antenna
The amplitude field pattern is the measured (calculated) and plotted electric (magnetic) field intensity, |E(θ, φ)| or |H(θ, φ)| at a constant (large) distance from the antennas

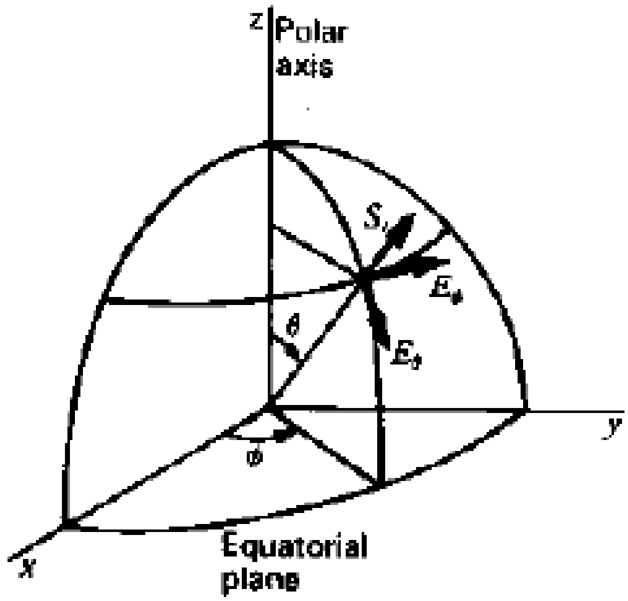


Fig 4.3: Relation of poynting vector s and 2 electric field components of a far field

ARRAY OF TWO POINT SOURCES

ARRAY is an assembly of antennas in an electrical and geometrical of such a nature that the radiation from each element add up to give a maximum field intensity in a particular direction& cancels in other directions. An important characteristic of an array is the change of its radiation pattern in response to different excitations of its antenna elements.

Here let us consider the different cases of two isotropic sources placed λ/2 apart with different scenarios.

1. **Obtain the field pattern for 2 isotropic point sources with equal amplitude and opposite phase. Assume distance between 2 sources is 'd'.**

<u>Sol :</u>

This case is identical with the previous but two sources are in opposite phase instead of same phase let the two sources 1 and 2 are located symmetrically with respect to origin of –ve coordinates consider a observation point p at distance 'r', the angle θ in measured clockwise from positive x-axis

if origin is considered as reference, the field from source 1 is related by (dr/2)cos θ and field from source 2 is advanced by

$$(dr/2) \cos \theta \text{ wr } dr = \beta d = 2\pi/2jEo * d \quad \text{.................... 1}$$

then total electric feald in the direction at a large distance r is given

$$E=2Eo\,[\exp(j*\Psi/2 - \exp(-j*\Psi/2)]$$

From which $$E=2JEo[(\exp(j*\Psi/2) - \exp(-j*\Psi/2))/2] \quad \text{................. 2}$$

j indicates the phase reversal of one source and it is not in portent

$$E=2jEo \sin(\Psi/2)$$

$$E=2jEo \sin((dr/2) \cos \theta \quad \text{.................. 3}$$

Normalize eq3 2jEo=1, for d=π

$$E=\sin((\pi/2) \cos \theta) \quad \text{....................4}$$

Since dr= $\beta d = 2\pi/\lambda * (\lambda/2) = \pi$

In above eq

For θ=0, E=1

θ=30, E=0.977

θ=60, E=0.70

θ=90, E=0

θ=120, E=0.70

θ=150, E=0.97

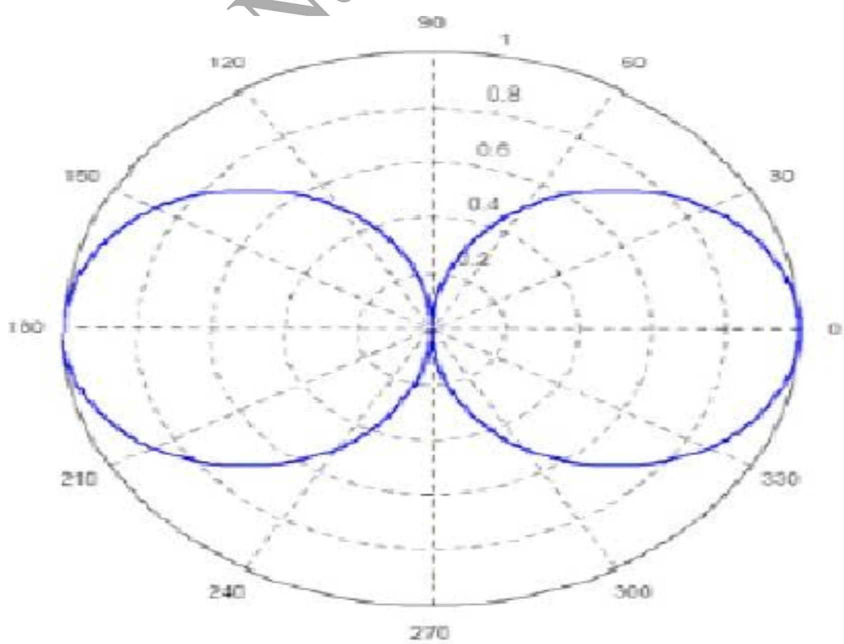
θ=180, E=-1

θ=210, E=-0.97

θ=240, E=-0.70

θ=270, E=0

θ=300, E=0.70



θ=330, E=0.97, θ=360, E=1

ARRAY OF 'n' ISOTROPIC POINT SOURCES

Uniformly excited equally spaced linear arrays Linear arrays of N-isotropic point sources of equal amplitude and spacing: An array is said to be linear if the individual elements of the array are spaced equally along a line and uniform if the same are fed with currents of equal amplitude and having uniform phase shift along the line

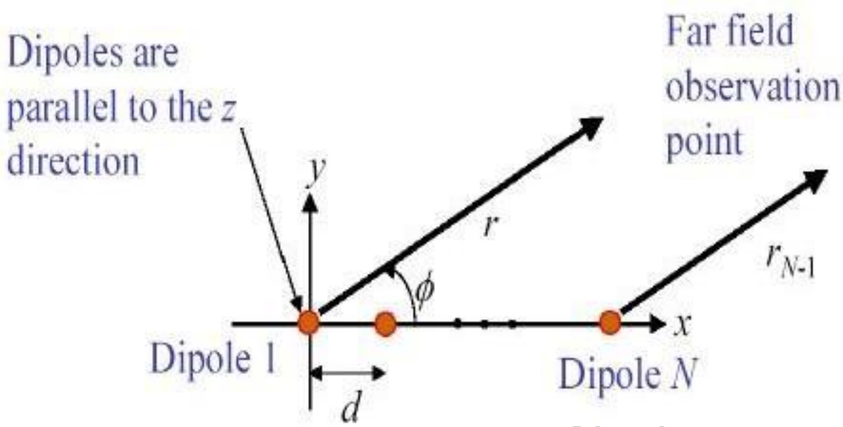


Fig.4.4 Linear arrays of N-isotropic point sources of equal amplitude and spacing:

The total field E at distance point in the direction of is given by

$$E=1+e^{j\psi}+ e^{j2\psi} +e^{j2\psi} +................................. + e^{j(n-1)\psi} \quad (1)$$

Where Ψ= total phase difference between adjacent source Ψ =dr*cos +δ =2π/λ*d*cosφ +δ

δ = phase difference of adjacent source

multipliying equation (1) by $e^{j\varphi}$

$$E\, e^{j\psi} =e^{j\psi} + e^{j2\psi} +e^{j3\psi} +................................ + e^{jn\psi} \quad (3)$$

Subtract (1)-(3) yields

$E(1- e^{j\psi}) = (1- e^{jn\psi})$ $E=1e^{jn\psi}/1- e^{j\psi}$

$E = e^{j(n-1)\psi/2} \{\sin(n\Psi/2)) /\sin(\Psi/2) \}$

If the phase is referred to the centre point of the array, then E redu ces to

$E=(\sin(n\Psi/2)) /\sin(\Psi/2)$

when Ψ=0 $E=\lim (\sin(n\Psi/2)) /\sin(\Psi/2)$

ψ 0, E =n=Emax

Ψ=0 E=Emax= n.............normalizing

$E_{norm} =E/E_{max} =(1/n)(\sin(n\Psi/2)) /\sin(\Psi/2)$

<u>CASE 1:</u> LINEAR BROAD SIDE ARRAY

An array is said to be broadside if the phase angle is such that it makes maximum radiation perpendicular to the line of array i.e. 90^0&270^0

For broad side array $\Psi=0$ & $\delta=0$

Therefore $\Psi = dr*\cos\Phi + \delta = \beta d\cos\Phi + 0 = 0$ $\Phi = \pm 90^0$

therefore $\Phi_{max} = 90^0$&270^0

Broadside array example for n=4 and d=λ/2

By previous results we have $\Phi_{max} = 90^0$&270^0

Direction of pattern maxima:

$$E=(1/n)(\sin(n\Psi/2))/\sin(\Psi/2)$$

This is maximum when numerator is maximum i.e. $\sin(n\Psi/2)=1$ $n\Psi/2=\pm(2k+1)\pi/2$ where k=0,1,2........
K=0 major lobe maxima
K=1 $n\Psi/2=\pm 3\pi/2$ $\Psi=\pm 3\pi/4$
Therefore $dr*\cos\Phi = 2\pi/\lambda*d*\cos\Phi = \pm 3\pi/4$ $\cos\Phi = \pm 3/4$

$\Phi = (\Phi_{max})_{minor\ lobe} = \cos^{-1}(\pm 3/4) = \pm 41.40$ or ± 138.60

At K=2, $\phi = \cos^{-1}(\pm 5/4)$ which is not possible

Direction of pattern minima or nulls

It occurs when numerator=0 i.e. $\sin(n\Psi/2)=0$ $n\Psi/2=\pm k\pi$
where k=1,2,3................................ now using condition $\delta=0$
$\Psi=\pm 2k\pi/n=\pm k\pi/2$ $dr*\cos\Phi = 2\pi/\lambda*d/2*\cos\Phi$
Substituting for d and rearranging the above term $\pi\cos\Phi=\pm k\pi/2$ $\cos\Phi=\pm k/2$
Therefore $\Phi_{min} = \cos^{-1}(\pm k/2)$

k=1, $\Phi_{min} = \cos^{-1}(\pm 1/2) = \pm 60^0$ or $\pm 120^0$

k=2, $\Phi_{min} = \cos^{-1}(\pm 2/2) = \pm 0^0$ or 180^0

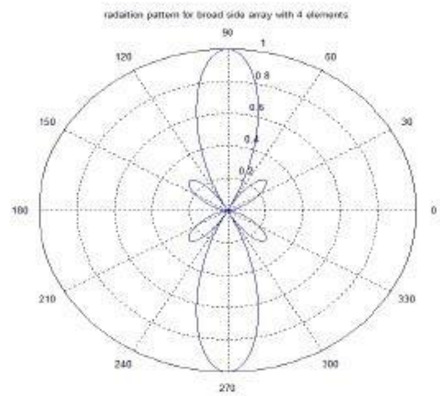


Fig.4.5 Radiation Array for Broadside Array with 4 Elements

From the pattern we see that

Beamwidth between first pair of nulls =BWFN=60 ^{0}

Half power beam width =BWFN/ 2=30^{0}

CASE2: END FIRE ARRAY

An array is said to be end fire if the phase angle is such that it makes maximum radiation in the line of array i.e. 0^{0}&180^{0}

For end fire array $\Psi=0$ & Φ =00 &1800

Therefore $\Psi = dr*\cos\Phi + \delta$ $\delta = -dr$

The above result indicates that for an end fire array the phase difference b/w sources is retarded progressively by the same amount as spacing b/w the sources in radians.
If $d= \lambda/2$ $\delta = -dr = -2\pi/\lambda \times \lambda/2 = -\pi$
The above result indicates that source 2 lags behind source1 by π radians.

End fire array example for n=4 and d=λ/2

Direction of maxima

Maxima occurs when $\sin(n\Psi/2)=1$
i.e. $\Psi/2 = \pm(2k+1)\pi/2$ where k=0,1,2........
$\Psi = \pm(2k+1)\pi/n$ $dr*\cos\Phi + \delta = \pm(2k+1)\pi/n$
$\cos\Phi = [\pm(2k+1)\pi/n - \delta]/dr$

Therefore $\Phi_{max} = \cos^{-1}\{[\pm(2k+1)\pi/n - \delta]/dr\}$
By definition For end fire array : $\delta = -dr = -2\pi/\lambda * d$
Therefore $\Phi_{max} = \cos^{-1}\{[\pm(2k+1)\pi/n - \delta]/(-2\pi/\lambda * d)\}$
For n=4, d=λ/2 dr=π after substituting these values in above equation & solving we get
$\Phi_{max} = \cos^{-1}\{[\pm(2k+1)/4 + 1\}$ Where k=0,1,2......

For major lobe maxima,

$\Psi = 0 = dr*\cos\Phi + \delta$
$= dr*\cos\Phi - dr$
$= dr(\cos\Phi - 1)$ $\cos\Phi_{m=1}$ there fore $\Phi_{m=00}$ or 1800
Minor lobe maxima occurs when k=1,2,3......
K=1 $(\Phi\ max)minor1 = \cos^{-1}\{[\pm(3)/4 + 1\}$
$= \cos^{-1}(7/4$ or $1/4)$ Since $\cos^{-1}(7/4)$ is not possible
Therefore $(\Phi\ max)minor1 = \cos^{-1}(1/4) = 75.5$

K=2 $(\Phi\ max)minor2 = \cos^{-1}\{[\pm(5)/4 + 1\}$
$= \cos^{-1}(9/4$ or $-1/4)$
Since $\cos^{-1}(9/4)$ is not possible
Therefore
$(\Phi\ max)minor1 = \cos^{-1}(-1/4) = 104.4$

Direction of nulls:

it occurs when numerator=0
i.e. $\sin(n\Psi/2) = 0$ $n\Psi/2 = \pm k\pi$
where k=1,2,3 .... Here $\Psi = dr*\cos\Phi + \delta = dr(\cos\Phi - 1)$ $dr = 2\pi/\lambda*\lambda/2 = \pi$
Substituting for d and n
$dr(\cos\Phi - 1) = \pm 2k\pi/n$ $\cos\Phi = \pm k/2 + 1$ therefore
Φ null $= \cos^{-1}(\pm k/2 + 1)$
k=1 , Φ null1 $= \cos^{-1}(\pm 1/2 + 1) = \cos^{-1}(3/2$ or $1/2)$
since $\cos^{-1}(3/2)$ not exist , Φ null1 $= \cos^{-1}(1/2) = \pm 60$ there fore
Φ null1 $= \pm 60$

k= 2,
Φ null2 $= \cos^{-1}(\pm 2/2 + 1)$
$= \cos^{-1}(2$ or $0)$
since $\cos^{-1}(2)$ not exist ,
Φ null2 $= \cos^{-1}(0) = \pm 90$ there fore Φ null2 $= \pm 90^0$
k=3, Φ null3 $= \cos^{-1}(\pm 3/2 + 1) = \cos^{-1}(5/2$ or $-1/2)$
since $\cos^{-1}(5/2)$ not exist , Φ null3 $= \cos^{-1}(-1/2) = \pm 120^0$ there fore, Φ null3 $= \pm 120^0$

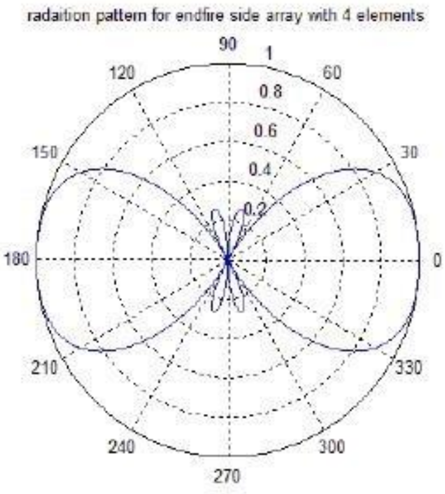


Fig.2.6 Radiation Array for End Fire Array with 5 Elements

OUTCOMES

- Able to calculate directivity for practical antennas by using the procedure
- Able to calculate major lobe minor lobe, HPBW, FNBW for two isotropic antennas, BSA, EFA different problems for given data

QUESTIONS

1. State and prove power theorem and its application.
2. Derive an expression for the power radiated from an isotropic point source with sine squared power pattern.
3. Eight point sources are spaced apart. They have a phase difference of $\pi/3$ between adjacent elements. Obtain the field pattern. Also find BWFN and HPBW.
4. Derive the expression for total field in case of two isotropic point sources with the same amplitude and equal phase. Plot the field pattern for two isotropic sources spaced apart.
5. Explain the principle of pattern multiplication.
6. Derive an expression and draw the field pattern for isotropic point sources of same amplitude and opposite phase. Also determine its maxima, minima and HPBW.
7. 4 isotropic point sources are placed apart. The power applied is with equal amplitude and a phase difference of $\pi/3$ between adjacent elements. Determine BWFN.
8. Derive the field equation for a linear array of n isotropic point sources of equal amplitude and spacing. Explain its operation as (a) broadside array (b) end fire array.

Further Readings

- **Antennas and Propagation for Wireless Communication Systems -** Sineon R Saunders, John Wiley, 2003.
- **Antennas and wave propagation -** G S N Raju: Pearson Education 2005.